Radical Equations

1. $\sqrt{m} = 6$
$\left(\sqrt{m}\right)^2 = \left(6\right)^2$
$m = 36$

2. $3\sqrt{x} = 21$
$\left(3\sqrt{x}\right)^2 = \left(21\right)^2$
$9x = 441$
$x = 49$

3. $2\sqrt{a} - 3 = 4$
$2\sqrt{a} = 7$
$\left(2\sqrt{a}\right)^2 = \left(7\right)^2$
$4a = 49$
$a = \dfrac{49}{4}$

4. $5 + 4\sqrt{x} = 11$
$4\sqrt{x} = 6$
$\left(4\sqrt{x}\right)^2 = \left(6\right)^2$
$16x = 36$
$x = \dfrac{36}{16} = \dfrac{9}{4}$

5. $\sqrt{5x} = 10$
$\left(\sqrt{5x}\right)^2 = \left(10\right)^2$
$5x = 100$
$x = 20$

6. $\sqrt{7x - 1} = 5$
$\left(\sqrt{7x - 1}\right)^2 = \left(5\right)^2$
$7x - 1 = 25$
$7x = 24$
$x = \dfrac{24}{7}$

7. $3\sqrt{2x + 3} = 12$
$\left(3\sqrt{2x + 3}\right)^2 = \left(12\right)^2$
$9(2x + 3) = 144$
$18x + 27 = 144$
$18x = 117$
$x = \dfrac{117}{18} = \dfrac{13}{2}$

8. $37 = 4\sqrt{2x + 6} - 3$
$40 = 4\sqrt{2x + 6}$
$\left(40\right)^2 = \left(4\sqrt{2x + 6}\right)^2$
$1600 = 16(2x + 6)$
$1600 = 32x + 96$
$1504 = 32x$
$x = \dfrac{1504}{32} = 47$

9. $\sqrt{\dfrac{5x - 3}{2}} = 2$
$\left(\sqrt{\dfrac{5x - 3}{2}}\right)^2 = \left(2\right)^2$
$\dfrac{5x - 3}{2} = 4$
$5x - 3 = 8$
$5x = 11$
$x = \dfrac{11}{5}$

$$7 = \sqrt{\dfrac{3m+1}{2}}$$

$$(7)^2 = \left(\sqrt{\dfrac{3m+1}{2}}\right)^2$$

10.
$$49 = \dfrac{3m+1}{2}$$

$$98 = 3m+1$$

$$97 = 3m$$

$$m = \dfrac{97}{3}$$

$$6 = \dfrac{3+\sqrt{2y-1}}{3}$$

$$18 = 3+\sqrt{2y-1}$$

$$15 = \sqrt{2y-1}$$

11. $\left(15\right)^2 = \left(\sqrt{2y-1}\right)^2$

$$225 = 2y-1$$

$$226 = 2y$$

$$113 = y$$

$$\dfrac{8-\sqrt{3-p}}{3} = 2$$

$$8-\sqrt{3-p} = 6$$

$$2 = \sqrt{3-p}$$

12. $\left(2\right)^2 = \left(\sqrt{3-p}\right)^2$

$$4 = 3-p$$

$$1 = -p$$

$$p = -1$$

$$\sqrt{p} - \sqrt{8} = 2\sqrt{50}$$

$$\sqrt{p} - \sqrt{2^3} = 2\sqrt{2 \cdot 5^2}$$

$$\sqrt{p} - 2\sqrt{2} = 2 \cdot 5\sqrt{2}$$

13.
$$\sqrt{p} = 12\sqrt{2}$$

$$\left(\sqrt{p}\right)^2 = \left(12\sqrt{2}\right)^2$$

$$p = 144 \cdot 2 = 288$$

$$3\sqrt{27} + 2\sqrt{x} = \sqrt{300}$$

$$3\sqrt{3^3} + 2\sqrt{x} = \sqrt{2^2 \cdot 5^2 \cdot 3}$$

$$3 \cdot 3\sqrt{3} + 2\sqrt{x} = 2 \cdot 5\sqrt{3}$$

$$9\sqrt{3} + 2\sqrt{x} = 10\sqrt{3}$$

14.
$$2\sqrt{x} = \sqrt{3}$$

$$\left(2\sqrt{x}\right)^2 = \left(\sqrt{3}\right)^2$$

$$4x = 3$$

$$x = \dfrac{3}{4}$$

$$7 - \sqrt{t^2 - 6t} = 3$$

$$-\sqrt{t^2 - 6t} = -4$$

$$\left(-\sqrt{t^2 - 6t}\right)^2 = \left(-4\right)^2$$

15. $t^2 - 6t = 16$

$$t^2 - 6t - 16 = 0$$

$$(t-8)(t+2) = 0$$

$$t = 8 \ \text{ or } -2$$

$$\sqrt{a^2 + 15a} - 10 = 0$$

$$\sqrt{a^2 + 15a} = 10$$

$$\left(\sqrt{a^2 + 15a}\right)^2 = \left(10\right)^2$$

16. $a^2 + 15a = 100$

$$a^2 + 15a - 100 = 0$$

$$(a+20)(a-5) = 0$$

$$a = -20 \ \text{ or } 5$$

$$\sqrt{2x^2 - x} - 3 = 2$$
$$\sqrt{2x^2 - x} = 5$$
$$\left(\sqrt{2x^2 - x}\right)^2 = (5)^2$$
$$2x^2 - x = 25$$

17. $2x^2 - x - 25 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-25)}}{2(2)}$$

$$x = \frac{1 \pm \sqrt{201}}{4}$$

$$\sqrt{7x + 11} = \sqrt{4x + 23}$$
$$\left(\sqrt{7x + 11}\right)^2 = \left(\sqrt{4x + 23}\right)^2$$

18. $7x + 11 = 4x + 23$

$3x = 12$

$x = 4$

$$\sqrt{2x - 5} = \sqrt{3x - 13}$$
$$\left(\sqrt{2x - 5}\right)^2 = \left(\sqrt{3x - 13}\right)^2$$

19. $2x - 5 = 3x - 13$

$-x = -8$

$x = 8$

$$\sqrt{x - 4}\sqrt{x + 4} = 3$$
$$\sqrt{x^2 - 16} = 3$$

20. $\left(\sqrt{x^2 - 16}\right)^2 = (3)^2$

$x^2 - 16 = 9$

$x^2 = 25$

$x = \pm 5$

$$\sqrt{x}\sqrt{x - 2} = 2\sqrt{2}$$
$$\sqrt{x(x - 2)} = 2\sqrt{2}$$
$$\sqrt{x^2 - 2x} = 2\sqrt{2}$$

21. $\left(\sqrt{x^2 - 2x}\right)^2 = \left(2\sqrt{2}\right)^2$

$x^2 - 2x = 4 \cdot 2$

$x^2 - 2x - 8 = 0$

$(x - 4)(x + 2) = 0$

$x = 4 \ \text{ or } \ -2$

$$\sqrt{3x + 13} = 2x - 3$$
$$\left(\sqrt{3x + 13}\right)^2 = (2x - 3)^2$$

22. $3x + 13 = 4x^2 - 12x + 9$

$0 = 4x^2 - 15x - 4$

$0 = (4x + 1)(x - 4)$

$x = 4 \ \text{ or } \ -\frac{1}{4}$

$$\sqrt{x^2 - 3x + 3} = x + 1$$

$$\left(\sqrt{x^2 - 3x + 3}\right)^2 = (x + 1)^2$$

23. $x^2 - 3x + 3 = x^2 + 2x + 1$

$-5x = -2$

$x = \dfrac{2}{5}$

$$\sqrt{5 + \sqrt{x}} = 4$$

$$\left(\sqrt{5 + \sqrt{x}}\right)^2 = (4)^2$$

24. $5 + \sqrt{x} = 16$

$\sqrt{x} = 11$

$\left(\sqrt{x}\right)^2 = (11)^2$

$x = 121$

$$6 = \sqrt{30 + \sqrt{x + 4}}$$

$$(6)^2 = \left(\sqrt{30 + \sqrt{x + 4}}\right)^2$$

$$36 = 30 + \sqrt{x + 4}$$

25. $6 = \sqrt{x + 4}$

$(6)^2 = \left(\sqrt{x + 4}\right)^2$

$36 = x + 4$

$32 = x$

$$\sqrt{3x + 1} - \sqrt{x + 4} = 1$$

$$\sqrt{3x + 1} = \sqrt{x + 4} + 1$$

$$\left(\sqrt{3x + 1}\right)^2 = \left(\sqrt{x + 4} + 1\right)^2$$

$$3x + 1 = x + 4 + 2\sqrt{x + 4} + 1$$

$$2x - 4 = 2\sqrt{x + 4}$$

26. $(2x - 4)^2 = \left(2\sqrt{x + 4}\right)^2$

$4x^2 - 16x + 16 = 4(x + 4)$

$4x^2 - 16x + 16 = 4x + 16$

$4x^2 - 20x = 0$

$4x(x - 5) = 0$

$x = 0$ or 5

$$\sqrt{x+2}+2=\sqrt{3x+4}$$
$$\left(\sqrt{x+2}\right)^2=\left(\sqrt{3x+4}-2\right)^2$$
$$x+2=3x+4-4\sqrt{3x+4}+4$$
$$-2x-6=-4\sqrt{3x+4}$$

27. $\left(-2x-6\right)^2=\left(-4\sqrt{3x+4}\right)^2$

$$4x^2+24x+36=16(3x+4)$$
$$4x^2+24x+36=48x+64$$
$$4x^2-24x-28=0$$
$$4(x-7)(x+1)=0$$
$$x=7 \text{ or } -1$$

$$\sqrt{x^2+3x+6}-\sqrt{x^2+3x-1}=1$$
$$\sqrt{x^2+3x+6}=\sqrt{x^2+3x-1}+1$$
$$\left(\sqrt{x^2+3x+6}\right)^2=\left(\sqrt{x^2+3x-1}+1\right)^2$$
$$x^2+3x+6=x^2+3x-1+2\sqrt{x^2+3x-1}+1$$
$$6=2\sqrt{x^2+3x-1}$$

28. $(6)^2=\left(2\sqrt{x^2+3x-1}\right)^2$

$$36=4(x^2+3x-1)$$
$$36=4x^2+12x-4$$
$$0=4x^2+12x-40$$
$$0=4(x+5)(x-2)$$
$$x=-5 \text{ or } 2$$

$$\sqrt[3]{4x-1}=-3$$
$$\left(\sqrt[3]{4x-1}\right)^3=(-3)^3$$

29. $4x-1=-27$

$$4x=-26$$
$$x=-\frac{26}{4}=-\frac{13}{2}$$

$$(3x-3)^{\frac{2}{3}}=9$$
$$\left((3x-3)^{\frac{2}{3}}\right)^{\frac{3}{2}}=(9)^{\frac{3}{2}}$$

30. $3x-3=\left(3^2\right)^{\frac{3}{2}}=3^3=27$

$$3x=30$$
$$x=10$$

$$\sqrt[5]{2x+6}=-2$$
$$\left(\sqrt[5]{2x+6}\right)^5=(-2)^5$$

31. $2x+6=-32$

$$2x=-38$$
$$x=-19$$

$$\sqrt[3]{3x-2}=\sqrt[3]{4x}$$
$$\left(\sqrt[3]{3x-2}\right)^3=\left(\sqrt[3]{4x}\right)^3$$

32. $3x-2=4x$

$$-x=2$$
$$x=2$$